

M.Sc. Sem II.

MPHYCC - 6

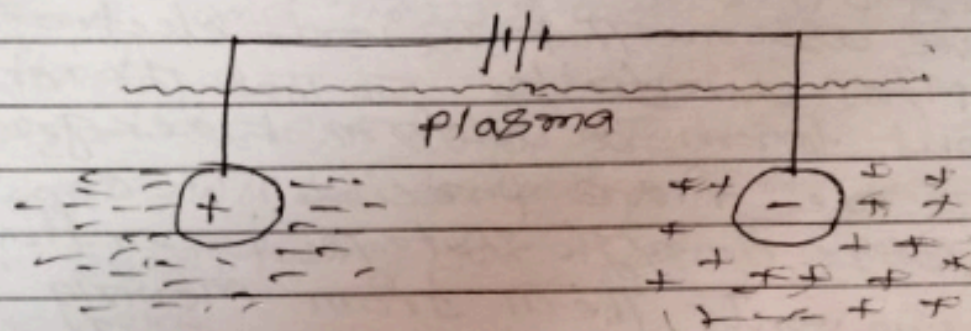
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Debye Shielding :-

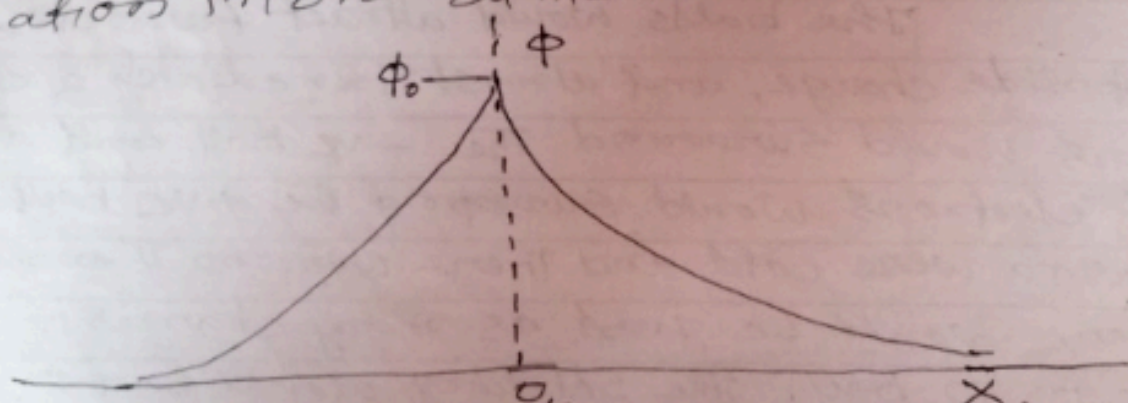
A fundamental characteristic of the behaviour of a plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside a plasma by inserting two charged balls connected to a battery.



The balls would attract particles of the opposite charge, and almost immediately a cloud of ions would surround the -ve ball and a cloud of electrons would surround the +ve ball. If the plasma were cold and there were no thermal motion, there would be just as many charges in the cloud as in the ball. The shielding would be perfect and no electric field would be present in the body of the plasma outside of the clouds. On the other hand, if the temp. is finite, those particles that are at the edge of the cloud, where the electric field is weak, have enough thermal

energy to escape from the electrostatic potential well. The edge of the cloud then occurs at the radius where the potential energy is approximately equal to the thermal energy kT of the particles, and the shielding is not complete: potentials of the order of kT/e can leak into the plasma and cause finite electric fields to exist there.

Let us compute the approximate thickness of such a charge cloud. Imagine that the potential ϕ on the plane $x=0$ is held at a value ϕ_0 by a perfectly transparent grid (fig. we wish to compute $\phi(x)$). For simplicity we assume that the ion-electron mass ratio M/m is infinite. So that the ions do not move but form a uniform background of +ve charge. To be more precise, we can say that M/m is large enough that the inertia of the ions prevents them from moving significantly on the time scale of the experiment. Poisson's equation in one dimension is,



Potential distribution near a grid in a plasma

$$\epsilon_0 \nabla^2 \phi = \epsilon_0 \frac{d^2 \phi}{dx^2} = -e(n_i - n_e) \quad (z=1) \quad \text{--- (1)}$$

If the density far away is n_∞ , we have

$$n_i = n_\infty$$

In the presence of a potential energy $q\phi$, the electron distribution function is

$$f(u) = A \exp\left[-\left(\frac{1}{2} mu^2 + q\phi\right)/kT_e\right]$$

It would not be worthwhile to prove this here. What this equation says is intuitively obvious: there are fewer particles at places where the potential energy is large, since not all particles have enough energy to get there. Integrating $f(u)$ over u , setting $q = -e$, and noting that $n_e(\phi \rightarrow 0) = n_\infty$, we find

$$n_e = n_\infty \exp(e\phi/kT_e)$$

This equation will be derived with more physical insight. Substituting for n_i and n_e in equation (1), we have

$$\epsilon_0 \frac{d^2\phi}{dx^2} = en_\infty \left\{ \left[\exp\left(\frac{e\phi}{kT_e}\right) \right] - 1 \right\}$$

In the region where $|e\phi/kT_e| \ll 1$, we can expand the exponential in a Taylor series

$$\epsilon_0 \frac{d^2\phi}{dx^2} = en_\infty \left[\frac{e\phi}{kT_e} + \frac{1}{2} \left(\frac{e\phi}{kT_e}\right)^2 + \dots \right] \quad (2)$$

No simplification is possible for the region near the grid where $|e\phi/kT_e|$ may be large. Fortunately, this region does not contribute much to the thickness of the cloud, because the potential falls very rapidly there. Keeping only the linear terms in equation (2), we have

$$\epsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{kT_e} \phi \quad (3)$$

Defining

$$\lambda_D \equiv \left(\frac{\epsilon_0 k T_e}{n e^2} \right)^{1/2} \quad \text{--- (4)}$$

where n stands for n_0 , we can write the solution of equⁿ (3) as

$$\phi = \phi_0 \exp(-|x|/\lambda_D) \quad \text{--- (5)}$$

The quantity λ_D called the Debye length, is a measure of the shielding distance or thickness of the sheath.

Note that as the density is increased, λ_D decreases, as ~~one~~ one would expect since each layer of plasma contains more electrons. Furthermore, λ_D increases with increasing kT_e without thermal agitation, the charge cloud would collapse to an infinitely thin layer. Finally, it is the electron temp. which is used in the definition of λ_D because the electrons, being more mobile than the ions, generally do the shielding by moving so as to create a surplus or deficit of $-ve$ charge. Only in special situations is ~~not~~ this not true.

The following are useful forms of equⁿ (4)

$$\lambda_D = 69 (T_e)^{1/2} \text{ m}, \quad T \text{ in } ^\circ\text{K} \quad \text{--- (4)}$$

$$\lambda_D = 7430 (kT_e/n)^{1/2} \text{ m}, \quad kT \text{ in eV} \quad \text{--- (6)}$$

we are now in a position to define "quasi-neutrality." If the dimensions L of a system are much larger than λ_D , then whenever local concentrations of charge arise or external potentials are introduced into the system. These are shielded out in a distance short compared with L , leaving the bulk of the plasma free of large electric potentials or fields.

outside of the sheath on the wall or on an obstacle, $\nabla^2 \phi$ is very small and n_i is equal to n_e , typically, to better than one part in 10^6 . It takes only a small charge imbalance to give rise to potentials of the order of kT/e . The plasma is "quasi-neutral"; i.e. neutral enough so that one can take $n_i \approx n_e \approx n$, where n is a common density called the plasma density, but so neutral that all the interesting electromagnetic forces vanish.

A criterion for an ionized gas to be a plasma is that it be dense enough that λ_D is much smaller than L .

The phenomenon of Debye shielding also occurs - in modified form - in single-species systems, such as the electron streams in klystrons and magnetrons or the proton beam in a cyclotron.